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EFFECT OF THERMOFLUID PARAMETERS ON COMBINED CONVECTION IN A TRIANGULAR WAVY CHANNEL

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ABSTRACT

The effect of Reynolds number on combined convective flow and heat transfer characteristics through a triangular wavy vertical channel is performed using the Galerkin weighted residual finite element method. The flow enters at the bottom and exits from the top surface. The wavy vertical walls are at constant temperature and the cold flow enters the channel at the bottom side. The numerical model is based on a 2D Navier-Stokes incompressible flow and energy equation. The effect of Reynolds number on flow and thermal fields is investigated. The variation of local Nusselt number along the vertical walls is also presented.

Keywords: Combined Convection, Wavy Channel, Finite Element Method.

1. INTRODUCTION

Mixed convection occurs if the effect of buoyancy forces on a forced flow or the effect of forced flow on a buoyant flow is significant. The governing nondimensional parameters for the description of mixed convection flows are Grashof number (*Gr*), Reynolds number (*Re*) and Prandtl number (*Pr*). It is necessary to study the heat and mass transfer from an irregular surface because irregular surfaces are often present in many applications such as micro-electronic devices, flat-plate solar collectors and flat-plate condensers in refrigerators, and geophysical applications (e.g., flows in the earth's crust), underground cable systems, electric machinery, cooling system of micro-electronic devices etc. In addition, roughened surfaces could be used in the cooling of electrical and nuclear components where the wall heat flux is known. One of the reasons why a roughened surface is more efficient in heat transfer is its capability to promote fluid motion near the surface. In this way a complex wavy surface, a sum of two or more wavy surfaces, is expected to promote a larger heat transfer rate than a single wavy surface. This complex geometry will promote a correspondingly complicated motion in the fluid near the surface. This motion is described by the nonlinear boundary-layer equations. This expectation is the basis of the current study even though only laminar mixed convection is studied. A vast amount of literature about convection along a sinusoidal wavy surface is available for different heating conditions and various kinds of fluids [1–3]. Recently Ashjaee et al. [4] have investigated the problem of free convection along a vertical wavy surface experimentally and numerically. The investigation was carried out for three different amplitude–wavelength ratios and Rayleigh number based on the length of the wavy surface ranging from $2.9*10^5$ to $5.8*10^5$. Results indicate that the frequency of the local heat transfer rate is the same as that of the wavy surface and the average heat transfer coefficient decreases as the amplitude-wavelength ratio increases. Chiu and Chou [5] studied the natural convection heat transfer along a vertical wavy surface in micropolar fluids. Chen and Wang [6-7] analyzed transient forced and free convection along a wavy surface in microfluids. Cheng [8] investigated coupled heat and mass transfer by natural convection flow along a wavy conical surface and vertical wavy surface in a porous medium. Shokouhmand and Abadi [9] calculated finite element analysis of mixed convection heat transfer through a vertical wavy isothermal channel where they considered sinusoidal wavy walls.

The aim of this study is to investigate the effects of parameters such as Grashof number and Reynolds number on flow and thermal fields through the triangular wavy channel. The local Nusselt number of vertical walls along the channel at the wide range of governing parameters (*Re*, *Gr*) is presented. The governing equations including continuity, Navier–Stokes and energy equations are solved numerically by using Galerkin finite element method.

2. MODEL SPECIFICATION

A two-dimensional vertical triangular wavy channel and related dimensionless boundary conditions are illustrated in Fig. 1. The channel consists of the mean diameter D, length L (L = 6D), amplitude A and the number of undulation (λ = 5). The cold fluid enters the channel with the conditions, $T = T_i$, $u = 0$ and $v = v_i$. The vertical walls of the channel are at constant temperature $T = T_h$.

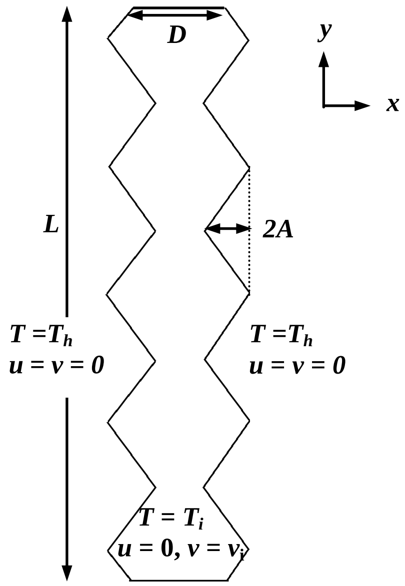


Fig 1. Physical model

3. MATHEMATICAL FORMULATION

A two-dimensional, steady, laminar, incompressible, mixed convection flow is considered within the channel and the fluid properties are assumed to be constant. The dimensionless equations describing the flow under Boussinesq approximation are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Gr}{Re^2}\theta \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

Here $Gr=\frac{g\beta\Delta T D^3}{\nu^2}$, $Re=\frac{v_i D}{\nu}$ and $Pr=\frac{\nu}{\alpha}$ are Grashof number, Reynolds number and Prandtl number respectively.

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{v_i},\ V=\frac{v}{v_i},\ P=\frac{p}{\rho v_i^2},\ \theta=\frac{(T-T_i)}{(T_h-T_i)}$$

The boundary conditions for the present problem are specified as follows:

at the inlet: $U = 0$, $V = 1$, $\theta = 0$

at the outlet: convective boundary condition $P = 0$

at all solid boundaries: $U = 0, V = 0, \theta = 1$

The local Nusselt number is calculated by the following expression : $Nu=-\frac{\partial\theta}{\partial n}$,where $\frac{\partial\theta}{\partial n}=\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$.

4. COMPUTATIONAL PROCEDURE

The momentum and energy balance equations are the combinations of mixed elliptic-parabolic system of partial differential equations that have been solved by using the Galerkin weighted residual finite element technique. The six node triangular element is used in this work for the development of the finite element equations. All six nodes are associated with velocities as well as temperature. Only three corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation. Firstly, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin's method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using reduced integration technique [10, 11] and Newton-Raphson method [12]. Finally, these linear equations are solved by applying Triangular Factorization method.

5. RESULTS AND DISCUSSION

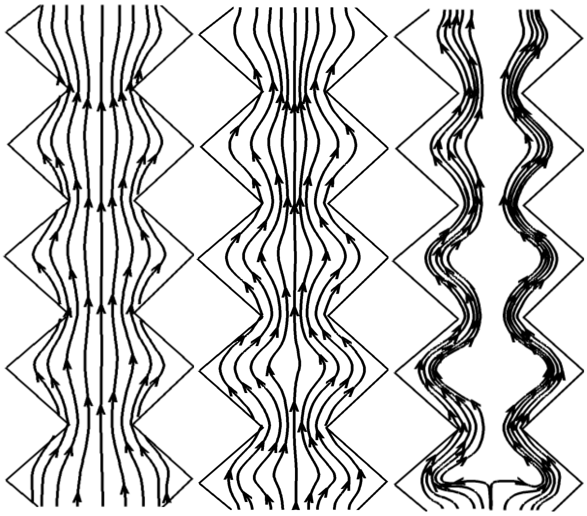
The mixed convection phenomenon inside a triangular wavy channel is influenced by different controlling parameters such as Reynolds number *Re*, Grashof number *Gr* and Prandtl number *Pr*. Analysis of the results is made through obtained streamlines, isotherms and local Nusselt number for different *Gr* and *Re*. Numerical solutions are obtained for values of $Gr = 10^4$-10^6, $Pr = 0.71$and $Re = 10$-500.

Fig. 2(a)-(c) shows the streamlines at $Re = 100$ and $Pr = 0.7$ for different values of Grashof number. Stream functions for higher values of Grashof number become more condensed at the regions near the side walls due to stronger convection effects at the middle of the channel.

Corresponding isothermal lines for various values of *Gr* at $Re = 100$ and $Pr = 0.7$ are shown in Fig. 3(a)-(c). With increment of Grashof number effect of buoyancy forces increases and excels the effect of forced convection which leads to decrement of peaks of isothermal lines at the middle of the channel. Increase of Grashof number also results in compression of isotherms at the inlet of the channel. Also at higher values of *Gr* the temperature of cold entering fluids will reach to T_h sooner than that of lower *Gr*.

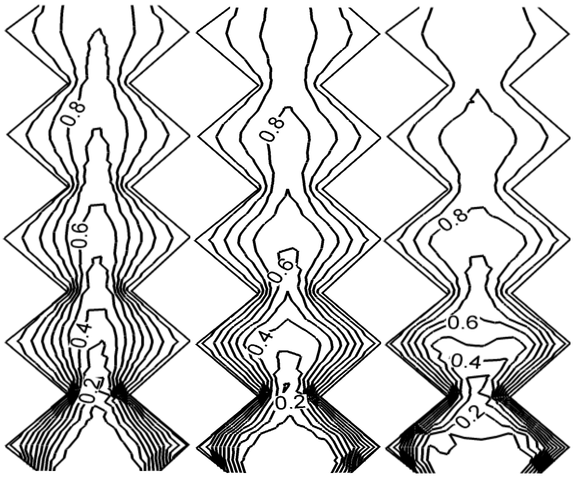
Fig. 4 illustrates the variation of local Nusselt number along the hot wall for different values of Grashof number for $Re = 100$ and $Pr = 0.7$. At the inlet of the channel the local Nusselt number is very large due to minimum thickness of boundary layer. With increasing *y*, the local

Nusselt number decreases due to decrement of temperature difference and becomes relative maximum at the peaks of hot walls due to sharp corners of walls which results in increment of compression of isotherms.



(a) (b) (c)

Fig 2. Streamlines at *Re* = 100, *Pr* = 0.7 for different values of *Gr*: (a) *Gr* = 10^4, (b) *Gr* = 10^5 and (c) *Gr* = 10^6



(a) (b) (c)

Fig 3. Isothermal lines at *Re* = 100, *Pr* = 0.7 for different values of *Gr*; (a) *Gr* = 10^4, (b) *Gr* = 10^5 and (c) *Gr* = 10^6

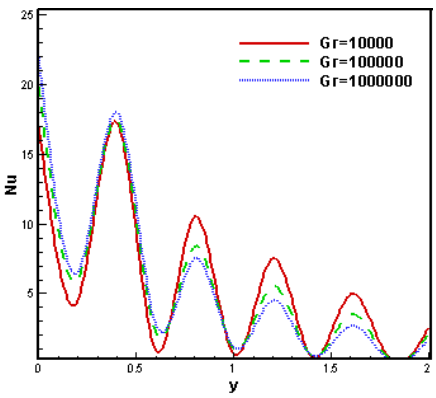
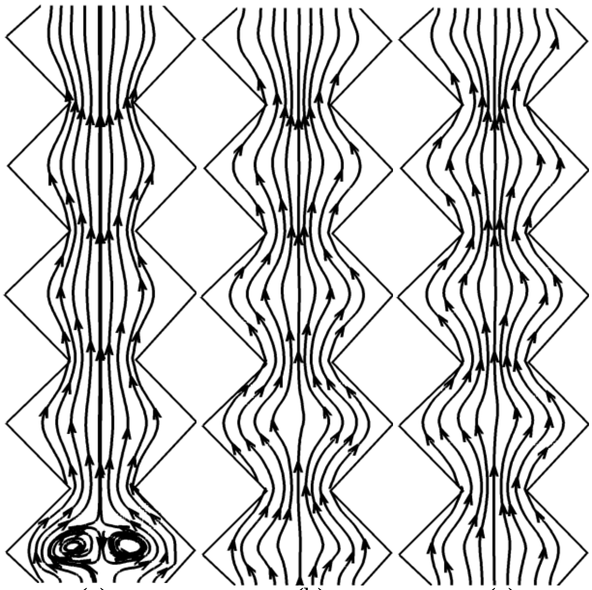
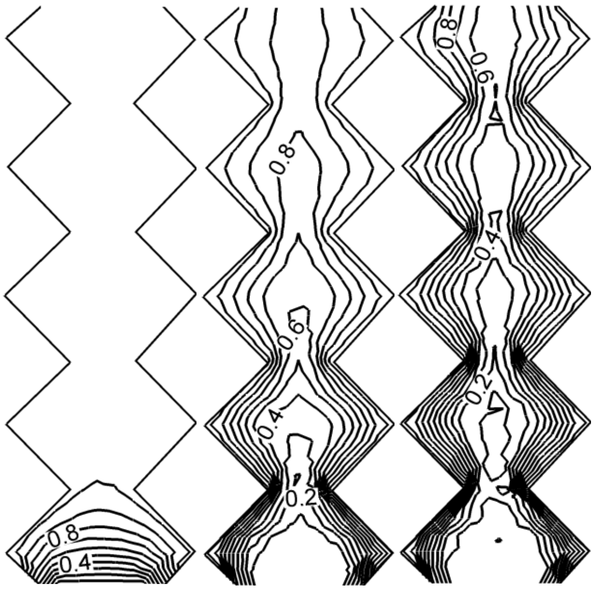


Fig 4. Variation of local Nusselt number along the wavy wall of the channel for different values of *Gr* at *Re* = 100 and *Pr* = 0.7.



(a) (b) (c)

Fig 5. Streamlines at *Gr* = 10^5, *Pr* = 0.7 for different values of *Re*; (a) *Re* = 10, (b) *Re* = 100 and (c) *Re* = 500



(a) (b) (c)

Fig 6. Isothermal lines at *Gr* = 10^5, *Pr* = 0.7 for different values of *Re*; (a) *Re* =10, (b) *Re* = 100 and (c) *Re* = 500

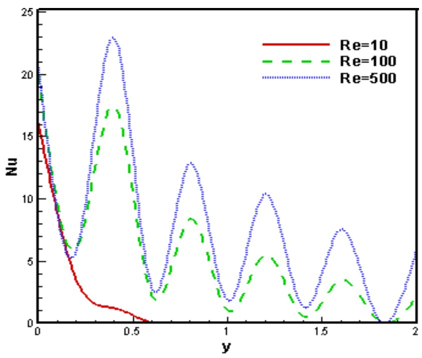


Fig 7. Variation of local Nusselt number along the wavy wall of the channel for different values of *Re* at *Gr* = 10^5 and *Pr* = 0.7.

Fig. 5(a)-(c) shows velocity field for different values of Reynolds number with $Gr = 10^5$ and $Pr = 0.7$. For $Re = 10$, two vortices produce due to uniform velocity of fluid at the inlet of the channel. Also plots for stream functions show that increment of Reynolds number results the vanishing of these vortices due to dominant force convection effect.

Fig. 6(a)-(c) demonstrates the temperature field for various Reynolds number while $Gr = 10^5$ and $Pr = 0.7$. For $Re = 10$ the temperature of cold entering fluid immediately reaches to T_h due to low velocity and momentum. With increasing Reynolds number the isotherms spreads all over the channel. It is also apparent that the compression of isotherms is increase due to sharp corners of side walls.

Fig. 7 plots the temperature profile for different values of Reynolds number with $Gr = 10^5$ and $Pr = 0.7$. At lower *Re* the temperature of the fluid reaches to T_h rapidly, therefore the local Nusselt number becomes zero. Increment of values of *Re* increases the local Nusselt number. Similar to Fig. 4 the variations of local Nusselt number has relative maximum due to curvature of hot walls. Also with increasing *y*, the local Nusselt number decreases due to decrement of temperature difference and increment of thickness of boundary layer.

6. CONCLUSION

Mixed convection heat transfer through a wavy vertical channel has been investigated numerically. The effects of parameters such as Grashof number and Reynolds number on flow and thermal fields are displayed. Results reveal that increasing Reynolds number reduced the secondary eddies at the inlet. At higher values of Reynolds number the temperature of the fluid reaches to the value T_h later. Elevation in *Gr* causes condensation of streamlines near the wavy walls and isotherms near the inlet. Local Nusselt number of hot walls decreases gradually through the channel and has relative maximum value at the peak of hot wavy wall for increasing both *Gr* and *Re*.

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7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
D	mean diameter of the channel	(m)
g	gravitational acceleration	(ms^{-2})
Gr	Grashof number	
L	length of the channel	(m)
Nu	Nusselt number	
p	dimensional pressure	(Nm^{-2})
P	non-dimensional	
Pr	Prandtl number	
Re	Reynolds number	
T	dimensional temperature	(K)
u, v	velocity components	(ms^{-1})
U, V	dimensionless components	
x, y	Cartesian coordinates	(m)
X, Y	non-dimensional coordinates	
Greek symbols		
α	thermal diffusivity	(m^2s^{-1})
β	thermal expansion coefficient	(K^{-1})
υ	kinematic viscosity of the fluid	(m^2s^{-1})
θ	non-dimensional temperature	
ρ	density of the fluid	(Kgm^{-3})
Subscripts		
h	hot	
i	inlet state	

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